Dealing with Uncertainty in Statics

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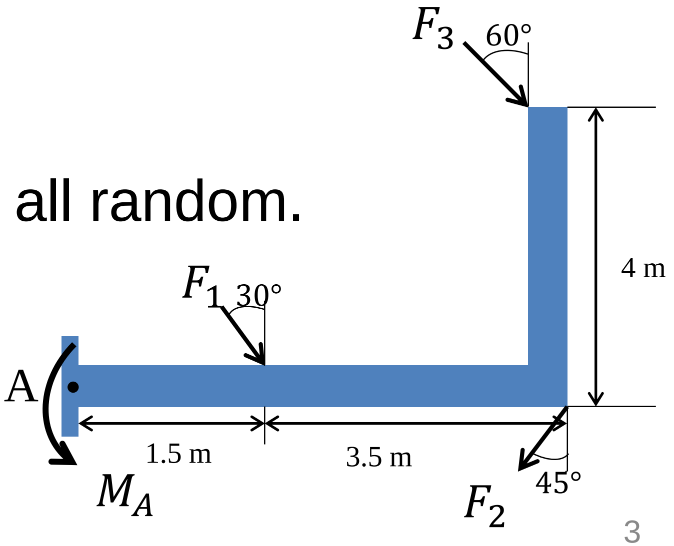
Outline

- Uncertainty in statics
- Why consider uncertainty
- Basics of uncertainty
- Uncertainty analysis in statics
- Examples
- Conclusions

Uncertainty in Statics: An Example

- Given: $M_A = 6000\ \mathrm{N \cdot m}$ counterclockwise, $F_1 = 350\ \mathrm{N}$, and $F_2 = 450\ \mathrm{N}$.
- Find: F_3
- Solution
  - $M_A = F_1 \cos 30°(1.5) + F_2 \cos 45°(5) + F_3 \cos 60°(5) + F_3 \sin 60°(4) = 0$
  - $F_3 = 663.0$ N
- Everything is modeled perfectly.

- In reality M_A, F_1, F_2 and the lengths are all random.
- So is F_3 - It fluctuates around 663.0 N.
- How do we know the fluctuation?



Where Does Uncertainty Come From?

- Manufacturing Imprecision
  - Dimensions of a structure
  - Material properties
- Environment
  - Loading
  - Temperature
  - Different user conditions
  - Wind, snow, etc.

Why Consider Uncertainty?

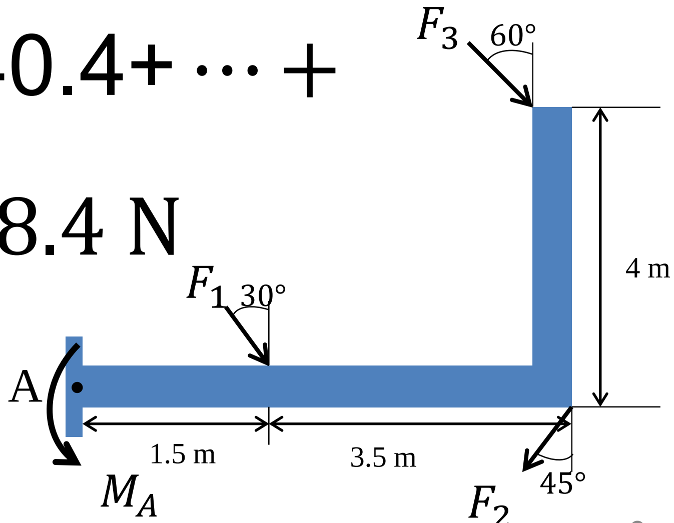
- We know the true solution
- We know the effect of uncertainty
- We can make more reliability decisions
- We can predict the chance of failure

If we consider uncertainties in the above example, we can design better cost-effective beams with sufficient reliability against failures with respect to strength, stiffness, etc.

How Do We Model Uncertainty?

- Measure X = F_1 ten times, we get samples
  X=(345.1,340.4,336.6,342.8,356.3,363.5, 351.9,349.3,342.4,355.7) N
- How do we use the samples?
- Average $\bar{X} = \frac{1}{10}$(345.1+340.4+ $\cdots +$ 342.4+ 355.7)= $\frac{1}{10}\sum X_i = 348.4$ N

(We can use excel)



How Do We Measure the Dispersion (Uncertainty)?

- X=(345.1,340.4,336.6,342.8,356.3,363.5,351.9,349.3,342.4,355.7) N

  For the deviation from $\bar{X}$, we could use $X_i - \bar{X}$ and $\frac{1}{N}\sum(X_i - \bar{X})$, $N = 10$
- But $\frac{1}{N}\sum(X_i - \bar{X}) = 0$.
- To avoid 0, we use $\frac{1}{N}\sum(X_i - \bar{X})^2$;
- To have the same unit as $\bar{X}$, we use $\sigma = \sqrt{\frac{1}{N}\sum(X_i - \mu)^2}$
- To make the estimate unbiased, we actually use

  standard deviation: $\sigma = \sqrt{\frac{1}{N-1}\sum(X_i - \mu)^2}$.
- Now we have $\sigma = 8.42$ N for $\bar{X} = 348.4$ N (We can use Excel)

More About Standard Deviation (std)

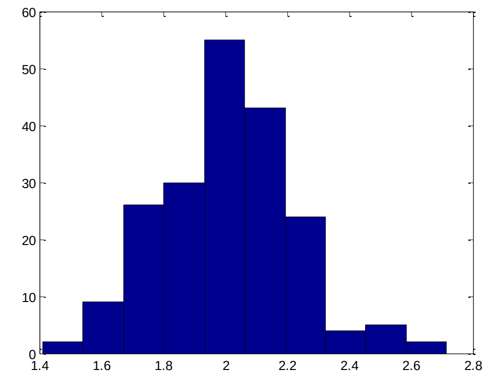
- It indicates how data spread around the average
- It is always non-negative
- High std means
  - High dispersion
  - High uncertainty
  - High risk

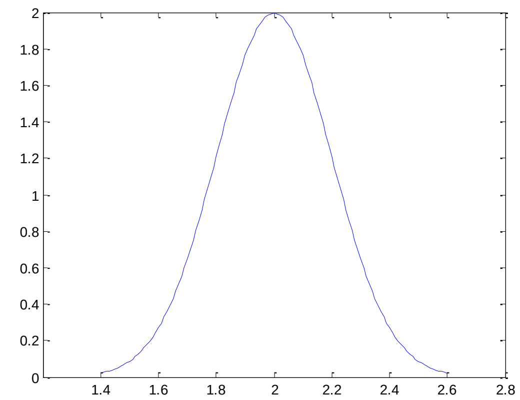
Probability Distribution

- With more samples, we can draw a histogram.

- If the vertical axis is the frequency and the number of samples is infinity, we get a probability density function (PDF) $f(x)$.
- The probability of $a \le X \le b$.

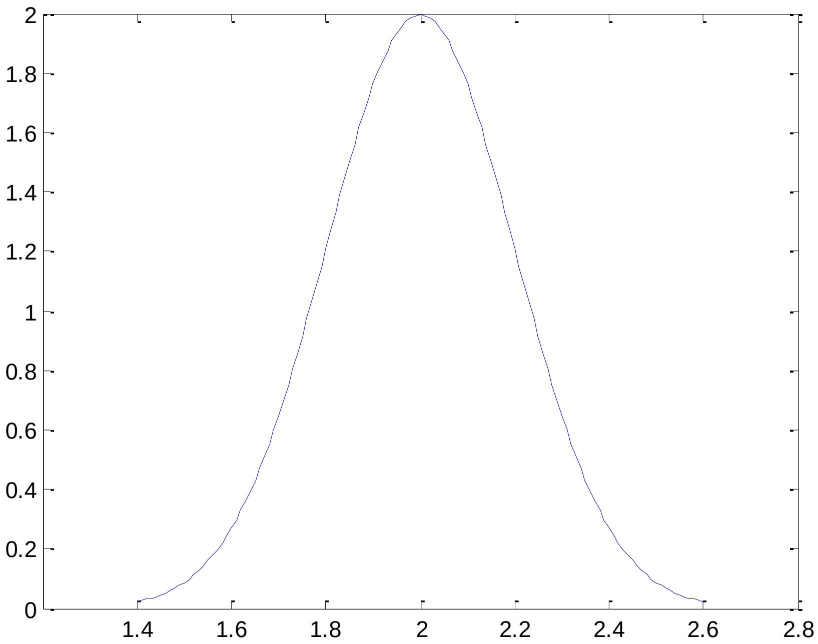
$$\Pr\{a \le X \le b\} = \int_a^b f(x)dx$$





Normal Distribution

- $X \sim N(\mu, \sigma^2)$
- μ – mean value, can be estimated by $\bar{X}$
- How to estimate σ has been discussed
- $F(x) = \Pr\{X < x\}$ is called cumulative distribution function (CDF)
- $\Pr\{a < X < b\} = F(b) - F(a)$
- $\Pr\{X > x\} = 1 - \Pr\{X \leq x\} = 1 - F(x)$



How to Calculate $F(x)$?

- Use Excel
  - NORMDIST(x,mean,std,cumulative)
  - Cumulative=true

Example 1

- The moment (in N·m) acting on a beam was recorded 20 times.
- Assume the moment is normally distributed.
- Questions

  - Mean =? Std = ?

  - What is the probability that the beam has a moment greater than 1192.3 N·m?

1	1198.5
2	1192.5
3	1188.1
4	1190.2
5	1195.1
6	1198.7
7	1196.3
8	1190.7
9	1193.9
10	1192.2
11	1191.5
12	1199.2
13	1197.7
14	1194.1
15	1196.9
16	1193.0
17	1197.5
18	1194.9
19	1199.1
20	1192.6

Solution: Use Excel

- $X =$ average moment
- $X \sim N\left(\mu, \sigma^2\right)$

  $\mu =$ 1194.635 N.m by AVERAGE

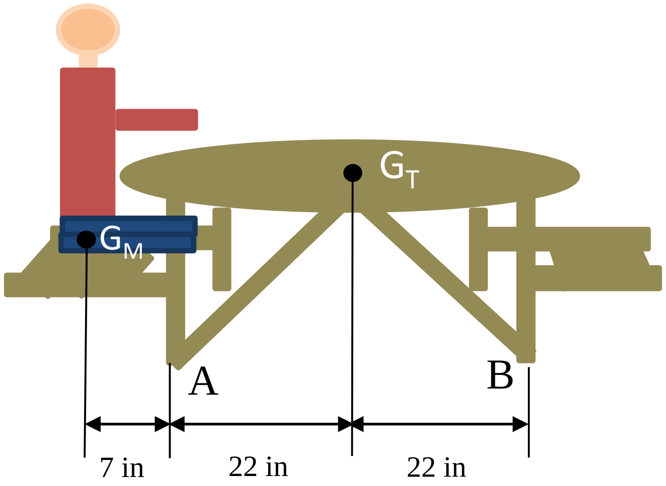
  $\sigma = 3.276\ \mathrm{N.m}$ by STDEV
- For $X > 1192.3\ \mathrm{N.m}$ $\Pr\{X > 1192.3\} = 1 - \Pr\{X \leq$

More Equations

- $X_i \sim N(\mu_i, \sigma_i^2)$
- $X_i\ (i = 1,2 \ldots n)\ \sigma_i^2$ independent
  - The occurrence of X_1 does not affect that of X_2
- $Y = c_0 + c_1 X_1 + \ \ldots + \ c_n X_n$
- $c_i\ (i = 1,2, \ldots, n)$ are constants
- Then $Y \sim N(\mu_y, \sigma_y^2)$
- $\mu_y = c_0 + c_1\mu_1 + c_2\mu_2 + \cdots + c_n\mu_n$
- $\sigma_y = \sqrt{c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \cdots + c_n^2\sigma_n^2}$

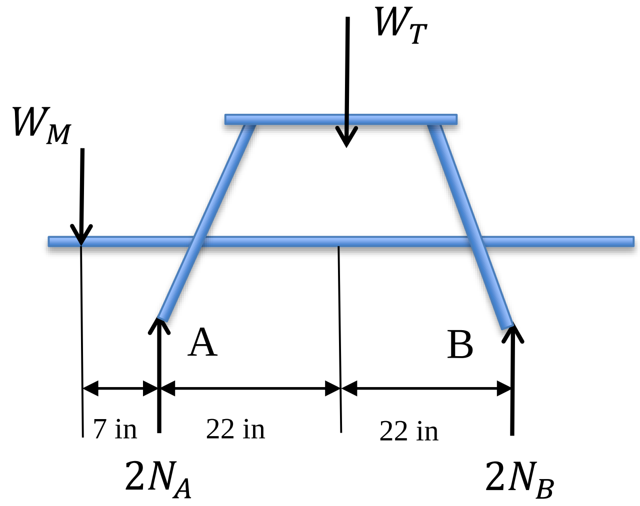
Example 2

A company plans to design a picnic table, which has a weight of W_T = 75 lb and a center of gravity at G_T. If a man weighing W_M= 215 lb has a center of gravity at G_M and sits down in the position shown, determine the vertical reaction at each of the two legs *B*. Will the table tip over? Neglect the thickness of the legs.



Example cont.

Solution



$$\sum M_A = 0 \quad (1)$$

$$2N_B(44) + W_M(7) - W_T(22) = 0 \quad (2)$$

$$N_B = 1.648 \text{ lb} \quad (3)$$

Since N_B is positive, the table will not tip over.

If Consider Uncertainty

- Due to uncertainty in users
  $W_M \sim N(215, 10^2)$ lb

  Due to manufacturing uncertainty
  $W_T \sim N(75, 1^2)$ lb

  W_M and W_T are independent.
- What is the probability that the table will tip over?

Solution:

$2N_B(44) + W_M(7) - W_T(22) = 0$

where $W_M \sim N(215, 10^2)$ lb and $W_T \sim N(75, 1^2)$ lb

Calculate μ_{N_B} and σ_{N_B}

$$\mu_{N_B} = \frac{\mu_{W_T}(22) - \mu_{W_M}(7)}{2*44} = 1.648 \text{ lb}$$

$$\sigma_{N_B} = \frac{1}{2*44}\sqrt{22^2{\sigma_{W_T}}^2 + 7^2{\sigma_{W_M}}^2} = 0.834 \text{ lb}$$

If the table tips over, $N_B < 0$. The probability is therefore

$P(N_B < 0) =$ NORMDIST$(0, 1.648, 0.834, \text{True}) = 0.0241$

The probability is about 2.4%.

Conclusions

- Uncertainty always exists in applications of statics.
- We can use a probability distribution, including its mean μ and standard deviation σ, to model uncertainty or randomness.
- Uncertainty may have a high impact on performance.
- If uncertainty is large, we should consider it during analysis and design in order to make better decisions.